Geometry Notes

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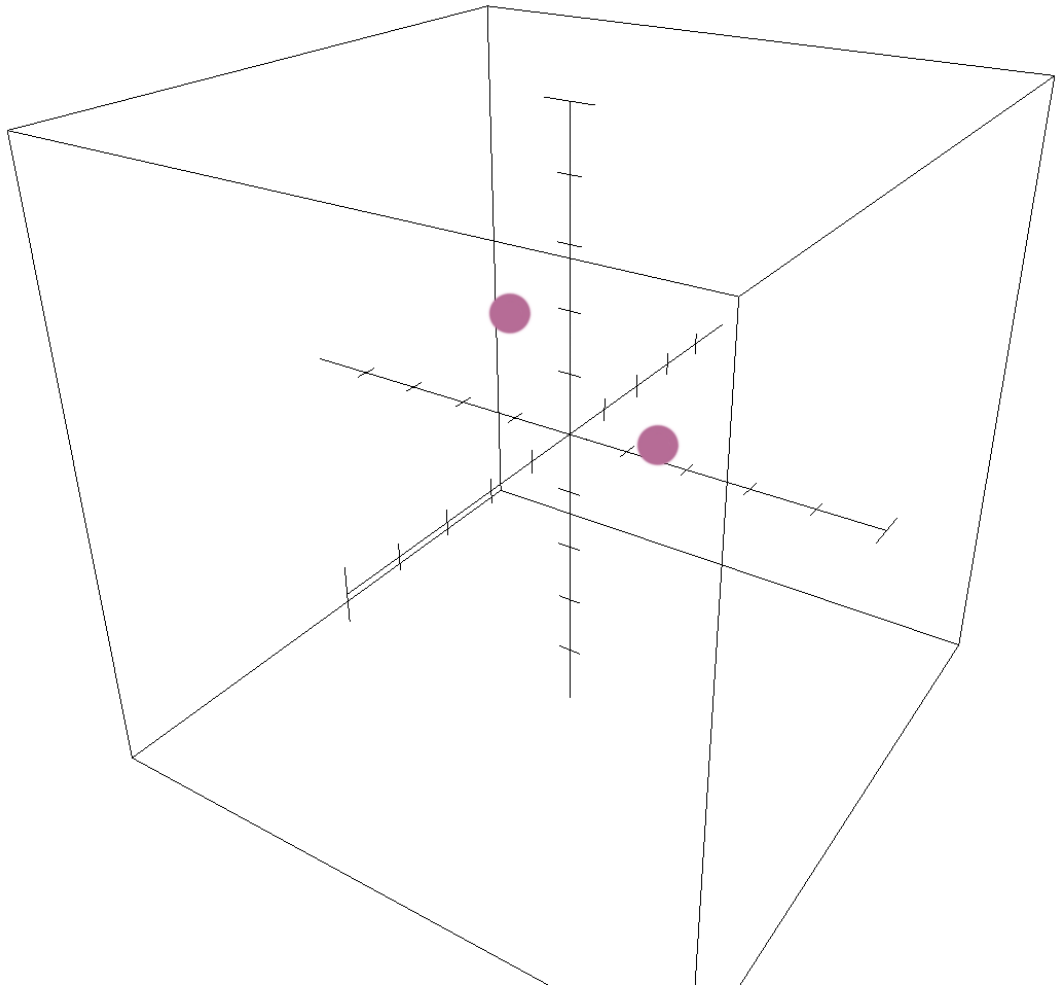
MATH

Distance between two points in three dimensions

In this lesson we'll look at points that are plotted three-dimensionally and how to find the distance between them.

Points

A point is a representation of a location in space. Its symbol is a single letter, and it's represented as a dot. When we're talking about a three-dimensional point, we have three locations: the x-axis, the y-axis and the z-axis. So a point is represented as (x, y, z). Here are two points A and B plotted in three-dimensional space:



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Distance formula

We can use the distance formula for three dimensions to find the length of the line segment that connects two points in three-dimensional space.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

where $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$.

Simplifying radicals

Since the distance formula takes the square root of a series of operations, it can be useful to know how to simplify radicals. Remember when you simplify a radical, that the radical has to have no perfect squares remaining under the radical symbol. For example,

$$\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$$

Now let's do an example of finding the distance between two points.

Example

Calculate the distance between Points B and C.

$$B = (4, -5, 8)$$

$$C = (1, -3, 2)$$

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We need to use the distance formula.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(4 - 1)^2 + (-5 - (-3))^2 + (8 - 2)^2}$$

$$d = \sqrt{(3)^2 + (-2)^2 + (6)^2}$$

$$d = \sqrt{9 + 4 + 36}$$

$$d = \sqrt{49}$$

$d = 7$ units

Now let’s try one where we’ll need to simplify the radical.

Example

Calculate the distance between Points A and D.

$$A = (3{,}9, -2)$$

$$D = (0, -5{,}4)$$

We need to use the distance formula.

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$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d = \sqrt{(3 - 0)^2 + (9 - (-5))^2 + (-2 - 4)^2}$$

$$d = \sqrt{(3)^2 + (14)^2 + (-6)^2}$$

$$d = \sqrt{9 + 196 + 36}$$

$$d = \sqrt{241} \approx 15.5 \text{ units}$$

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